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On the Perspective Transformation for Efficient Relay Placement in Wireless Multicast Networks

Quoc-Tuan Vien, Huan X. Nguyen, Balbir Barn and Xuan Nam Tran

***Abstract*—This letter investigates the relay placement problem in wireless multicast networks consisting of multiple sources, relays and destinations. The data transmission from the sources to the destinations is carried out via the relays employing physical-layer network coding technique. Hybrid automatic repeat request protocol with incremental redundancy is applied for reliable communication. Particularly, considering a general setting of nodes in irregularly-shaped network, an efficient relay placement algorithm is proposed based on perspective transformation technique to find optimal relay positions for minimising either the total energy consumption or the total delay in the whole network. The proposed algorithm not only helps reduce the relay searching complexity but also facilitates the relay placement for optimising networks of any shape.**

***Index Terms*—Relay placement, HARQ-IR, network coding, perspective transformation, wireless multicast network.**

I. Introduction

Multicasting has attracted growing interest in wireless communications [1]. Exploiting the broadcast nature of wireless medium, information data can be delivered from a sender to multiple intended receivers. With the advances of wireless communications, the demands of high-throughput and high-reliability multicasting are crucial, especially for remote users, e.g. cell-edge users in cellular networks. Recently, relaying techniques have been proposed to employ intermediate nodes or relay nodes not only for coverage enhancement but also for service quality improvement [2]. As an attempt to improve throughput, network coding (NC) has been applied at relay nodes to significantly improve the throughput of wireless networks by performing algebraic linear/logic operations on received packets [3].

Recently, relay placement problem has been extensively investigated in the literature (e.g. [4]–[6]). In [4], the relay position optimisation was proposed to improve diversity gain of unbalanced decode-and-forward relay networks. The optimal relay placement problem was investigated in [5] for amplify-and-forward relay networks. In [6], the relay positioning has been investigated for a wireless butterfly network consisting of two sources, a relay and two destinations. However, for general relay-assisted wireless multicast networks (RAWMNs) employing hybrid automatic repeat request with incremental redundancy (HARQ-IR) and NC protocols, the relay placement optimisation for minimising either transmission delay or energy consumption has not been investigated, especially when the sources and destinations are located in irregular shapes and may be varied over time.

In this letter, we investigate the relay placement problem for energy-efficient and reliable relaying in irregularly-shaped RAWMNs consisting of multiple sources, relays and destinations. HARQ-IR protocol is employed to guarantee the reliability of all communication links and physical-layer NC (PNC) protocol [3] is applied at the relays. Firstly, we derive the expression of energy consumption and delay for the HARQ-IR protocol with PNC in RAWMNs. The derived expressions enable us to optimise the relay placement for either minimum energy consumption or minimum delay in the whole network. Particularly, considering the general scenario of irregular node positions, we propose a novel optimal relay placement in RAWMNs by applying perspective transformation[1] to perform a spatial transformation of the 'real' node positions in an irregular quadrilateral to 'virtual' node positions in a rectangle. It is shown that the node mapping maintains the roles and functionalities of the nodes, and thus can be exploited to simplify the search for the 'real' optimal relay placement given the constraints on power allocation and location of the sources and destinations. The novelty of our work is that the determined virtual relay positions in the rectangle facilitate the searching of their real positions in any irregularly-shaped RAWMNs by simply finding the corresponding mapping matrix between shapes.

II. System Model and Relay Placement Problems

The system model of a RAWMN under investigation consists of N_s sources, N_r relays and N_d destinations. We consider a half-duplex RAWMN where all nodes can not simultaneously transmit and receive data. Without loss of generality, let us assume the k-th relay, $k = 1, 2, \ldots, N_r$ (i.e. $\mathcal{R}_k$), assists the data transmission from a group of $N_{s,k}$ sources (i.e. $\{\mathcal{S}_{k,1}, \mathcal{S}_{k,2}, \ldots, \mathcal{S}_{k,N_{s,k}}\} \triangleq \mathcal{S}_k^{(N_{s,k})}$) to a group of $N_{d,k}$ destinations (i.e. $\{\mathcal{D}_{k,1}, \mathcal{D}_{k,2}, \ldots, \mathcal{D}_{k,N_{d,k}}\} \triangleq \mathcal{D}_k^{(N_{d,k})}$). The indices of nodes are determined based upon their vertical axis values in a decreasing order, i.e. the node located higher has a lower index. The relay-aided transmission is basically realised in two time slots as follows: In the first time slot, $\mathcal{S}_{k,i_k}$, $i_k =$

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[1]The perspective transformation was initially proposed for image matching (e.g. in [7], [8]).

$1, 2, \ldots, N_{s,k}$, sends data to $\mathcal{R}_k$ and the corresponding $\mathcal{D}_{k,i'_k}$, $i'_k = 1, 2, \ldots, N_{d,k}$ via direct links. Then, $\mathcal{R}_k$ carries out PNC on the received signals before broadcasting the combined signal to all $\mathcal{D}_k^{(N_{d,k})}$ in the second time slot. Using the PNC technique, $\mathcal{D}_k^{(N_{d,k})}$ can decode all multicasting data from $\mathcal{S}_k^{(N_{s,k})}$. All channel links are assumed to suffer from quasi-static Rayleigh block fading and path loss with a pathloss exponent $\nu = 3$. For simplicity, it is also assumed that there is no interference caused by non-intended nodes in the RAWMN.

In this work, the positions of N_s sources and N_d destinations are assumed to be fixed in a two-dimensional plane while the positions of N_r relays vary in a convex set $\mathfrak{S}_T$ having its boundary formed by all the source and destination points[2]. Let us denote $\mathfrak{S}_k$, $k = 1, 2, \ldots, N_r$, as the convex set generated by points $\{\mathcal{S}_{k,1}, \mathcal{S}_{k,2}, \ldots, \mathcal{S}_{k,N_{s,k}}\}$ and $\{\mathcal{D}_{k,1}, \mathcal{D}_{k,2}, \ldots, \mathcal{D}_{k,N_{d,k}}\}$ which are in supporting region of $\mathcal{R}_k$. Then, we have

$$\mathfrak{S}_T \supseteq \bigcup_{k=1}^{N_r} \mathfrak{S}_k \tag{1}$$

Let (x_A, y_A), $A \in \{\{\mathcal{S}_i\}, \{\mathcal{R}_k\}, \{\mathcal{D}_j\}\}$, denote the coordinate values of a point $\mathcal{A}$. Also, let $\mathtt{D}_k$ and $\mathtt{E}_k$, $k = 1, 2, \ldots, N_r$, denote effective delay (ED) [sec/bit/Hz] and energy per bit (EB) [Joules/bit/Hz], respectively, in a subset $\mathfrak{S}_k$ [6]. The relay placement problems for either minimum total ED or minimum total EB in RAWMNs can thus be written by

$$\min_{(\{x_{\mathcal{R}_k}\}, \{y_{\mathcal{R}_k}\})} \sum_{k=1}^{N_r} \mathtt{D}_k, \tag{2}$$

$$\min_{(\{x_{\mathcal{R}_k}\}, \{y_{\mathcal{R}_k}\})} \sum_{k=1}^{N_r} \mathtt{E}_k, \tag{3}$$

respectively, where $(x_{\mathcal{R}_k}, y_{\mathcal{R}_k}) \in \mathfrak{S}_k$. It is assumed that there is no cooperation between relays, between sources and between destinations. Therefore, the above problems can be solved by separately treating each subset $\mathfrak{S}_k$.

III. Energy Consumption and Delay in RAWMNs

In this section, energy consumption and delay of HARQ-IR protocol for reliable data multicast in a RAWMN are derived. PNC protocol [3] is applied at relays to help forward data from multiple sources to multiple destinations.

Consider a specific $\mathcal{R}_k$, $k = 1, 2, \ldots, N_r$, who assists the data transmission from $\mathcal{S}_k^{(N_{s,k})}$ to $\mathcal{D}_k^{(N_{d,k})}$. In the first time slot, the number of transmissions required at $\mathcal{S}_k^{(N_{s,k})}$ to correctly send the data to $\mathcal{R}_k$ can be determined using the channel capacity bound [9, Lemma 1, eq. (6)], [10, Theorem 15.3.6, eq. (15.132)] as follows

$$\tau_{S_k^{(N_{s,k})} R_k} = \min \left\{ \tau \Big| \bigcap_{\Psi_k \subseteq \Omega_k} \left\{ \sum_{i=1}^{\tau} \log \left(1 + \sum_{\rho_k \in \Psi_k} [\gamma_{\rho_k R_k}]_i \right) > \sum_{\rho_k \in \Psi_k} r_{\rho_k} \right\} \right\}, \tag{4}$$

where Ω_k denotes the entire set generated by $\mathcal{S}_k^{(N_{s,k})}$, Ψ_k denotes the subset of Ω_k, ρ_k denotes an element in Ψ_k, $\gamma_{\rho_k R_k}$ denotes the signal-to-noise ratio (SNR) of the transmission link $\rho_k \to \mathcal{R}_k$ and r_{ρ_k} denotes the transmission rate at ρ_k. Here, $\log(\cdot)$ and $[a]_i$ are binary logarithm function and the i-th realisation of a random variable a, respectively. Also, in the first time slot, the number of transmissions required at $\mathcal{S}_{k,i_k}$ to send data to $\mathcal{D}_{k,i'_k}$, $i_k = 1, 2, \ldots, N_{s,k}$, $i'_k = 1, 2, \ldots, N_{d,k}$, is computed by

$$\tau_{S_{k,i_k} D_{k,i'_k}} = \min \left\{ \tau \Big| \sum_{i=1}^{\tau} \log \left(1 + [\gamma_{S_{k,i_k} D_{k,i'_k}}]_i \right) > r_{i_k} \right\}, \tag{5}$$

where $\gamma_{S_{k,i_k} D_{k,i'_k}}$ denotes the SNR of the transmission link $\mathcal{S}_{k,i_k} \to \mathcal{D}_{k,i'_k}$. The total number of transmissions required at $\mathcal{S}_{k,i_k}$ is thus given by

$$\tau_{S_{k,i_k}} = \max\{\tau_{S_k^{(N_{s,k})} R_k}, \tau_{S_{k,i_k} D_{k,i'_k}}\} \tag{6}$$

and the total number of transmissions in the first time slot is

$$\tau_{k_1} = \max\{\tau_{S_k^{(N_{s,k})} R_k}, \max_{i_k, i'_k}\{\tau_{S_{k,i_k} D_{k,i'_k}}\}\}. \tag{7}$$

In the second time slot, $\mathcal{R}_k$ encodes the superimposed packet using NC technique and then broadcasts the encoded packets to all $\mathcal{D}_k^{(N_{d,k})}$. The number of transmissions required at $\mathcal{R}_k$ to transmit data to $\mathcal{D}_{k,i'_k}$, $i'_k = 1, 2, \ldots, N_{d,k}$, can be similarly determined by

$$\tau_{R_k D_{k,i'_k}} = \min \left\{ \tau \Big| \sum_{i=1}^{\tau} \log \left(1 + [\gamma_{R_k D_{k,i'_k}}]_i \right) > r_{i'_k} \right\}, \tag{8}$$

where $\gamma_{R_k D_{k,i'_k}}$ is the SNR of the transmission link $\mathcal{R}_k \to \mathcal{D}_{k,i'_k}$. Therefore, the total number of transmissions in the second time slot is

$$\tau_{k_2} = \max_{i'_k}\{\tau_{R_k D_{k,i'_k}}\}. \tag{9}$$

Overall, following the same approach in [6], the total ED and EB of the HARQ-IR protocol in a RAWMN are given by

$$\mathtt{D} = \sum_{k=1}^{N_r} \frac{\tau_{k_1} + \tau_{k_2}}{\sum_{i_k} r_{i_k}}, \tag{10}$$

$$\mathtt{E} = \sum_{k=1}^{N_r} \frac{\sum_{i_k} P_{S_{k,i_k}} \tau_{S_{k,i_k}} + P_{R_k} \tau_{k_2}}{\sum_{i_k} r_{i_k}}, \tag{11}$$

where $P_{S_{k,i_k}}$ and P_{R_k}, $i_k = 1, 2, \ldots, N_{s,k}$, denote the transmission power at $\mathcal{S}_{k,i_k}$ and $\mathcal{R}_k$, respectively.

IV. Relay Placement with Perspective Transformation

In this section, we propose a novel relay placement algorithm to find optimal relay positions for either minimising ED or minimising EB of the HARQ-IR protocol in RAWMNs. In particular, we consider the general scenario when sources and destinations are irregularly located in a two-dimensional plane. First, let us briefly explain the principle of perspective transformation in image processing [8], which is then exploited for solving the relay placement problem.

[2]The assumption of the convex set implies the necessity of the relays in the RAWMN.

A. Perspective Transformation for Image Mapping

In order to map between two quadrilaterals, a perspective transformation or projective non-affine mapping with bilinear interpolation can be employed as follows: Given four 2-dimensional points A, B, C and D of an irregular quadrilateral located at (x_A, y_A), (x_B, y_B), (x_C, y_C) and (x_D, y_D), respectively. We can map the shape $ABCD$ to another quadrilateral $A'B'C'D'$ having four 2-dimensional points A', B', C' and D' located at $(x_{A'}, y_{A'})$, $(x_{B'}, y_{B'})$, $(x_{C'}, y_{C'})$ and $(x_{D'}, y_{D'})$, respectively. This mapping is realised via an 4×4 mapping matrix $\mathbf{M}$, which can be found by solving the following equation [8]:

$$\begin{pmatrix} 1 & x_A & y_A & x_A y_A \\ 1 & x_B & y_B & x_B y_B \\ 1 & x_C & y_C & x_C y_C \\ 1 & x_D & y_D & x_D y_D \end{pmatrix} \mathbf{M} = \begin{pmatrix} 1 & x_{A'} & y_{A'} & x_{A'} y_{A'} \\ 1 & x_{B'} & y_{B'} & x_{B'} y_{B'} \\ 1 & x_{C'} & y_{C'} & x_{C'} y_{C'} \\ 1 & x_{D'} & y_{D'} & x_{D'} y_{D'} \end{pmatrix} \quad (12)$$

It has been shown in [7], [8] that perspective transformation is planar mapping and thus both forward and inverse mapping are unique. Also, the lines connecting nodes are shown to be preserved in all orientations.

B. Relay Placement Algorithm

Taking into account the node location, it is intuitively observed that the relay placement problem is hard to solve for the general setting of node positions, especially when the locations of sources and destinations may vary over time. Exploiting the properties of perspective transformation, we can map the nodes in the irregularly-shaped RAWMNs to the nodes in a rectangle, namely virtual nodes, using (12). Then, we can easily find the optimal placement of virtual relays in the rectangular region to minimise either ED or EB (see (10) and (11)) using the same approach as in [6]. The real optimal positions of the relays can be thus determined by an inverse mapping. Accordingly, the problem is simply to compute the mapping matrices with respect to the location of sources and destinations.

For convenience, let us divide the entire set $\mathfrak{S}_T$ into N_r subsets with respect to N_r relays (see (1)) and consider a specific subset $\mathfrak{S}_k$, $k = 1, 2, \ldots, N_r$. The proposed relay placement algorithm for a general RAWMN can be realised as follows:

- *Step 1*: Map the boundary of $\mathfrak{S}_k$ to a rectangle, namely $\mathfrak{S}'_k$, by finding a mapping matrix $\mathbf{M}$ as in (12).
- *Step 2*: Find virtual positions of remaining sources and destinations in $\mathfrak{S}'_k$.
- *Step 3*: Find virtual relay position (x'_{R_k}, y'_{R_k}) in $\mathfrak{S}'_k$ for either minimising ED or minimising EB given by (10) and (11), respectively.
- *Step 4*: Find real relay position in $\mathfrak{S}_k$ by inverse mapping.

The relay placement algorithm is summarised in Algorithm 1. It can be observed that the proposed algorithm only requires the perspective transformation and determination of the optimal relay positions in a particular rectangle. This means that, based on the determined virtual relay positions in a rectangle (i.e. Step 3), we can find the real relay positions in any irregularly-shaped RAWMNs by simply finding the mapping matrix $\mathbf{M}$ between two shapes in Steps 1 and 2.

Algorithm 1 Proposed relay placement algorithm

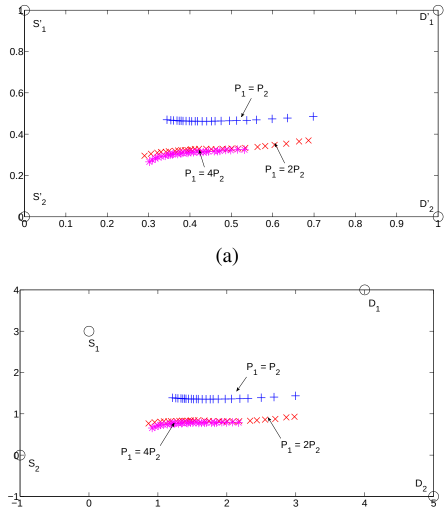
for $k = 1$ to N_r **do**
  $\mathfrak{S}_k \leftarrow \{\mathcal{S}_{k,1}, \mathcal{S}_{k,2}, \ldots, \mathcal{S}_{k,N_{s,k}}, \mathcal{D}_{k,1}, \mathcal{D}_{k,2}, \ldots, \mathcal{D}_{k,N_{d,k}}\}$
  Step 1: Map the boundary of $\mathfrak{S}_k$ to a rectangle $\mathfrak{S}'_k$:
   $(\mathcal{S}'_{k,1}\mathcal{S}'_{k,N_{s,k}}\mathcal{D}'_{k,1}\mathcal{D}'_{k,N_{d,k}}) \leftarrow (\mathcal{S}_{k,1}\mathcal{S}_{k,N_{s,k}}\mathcal{D}_{k,1}\mathcal{D}_{k,N_{d,k}})$
  Find mapping matrix $\mathbf{M}$ using (12).
  Step 2: Find virtual positions of remaining nodes in $\mathfrak{S}'_k$:
  for $i = 2$ to $N_{s,k} - 1$ **do**
   $[1, x_{S'_{k,i}}, y_{S'_{k,i}}, x_{S'_{k,i}} y_{S'_{k,i}}] \leftarrow [1, x_{S_{k,i}}, y_{S_{k,i}}, x_{S_{k,i}} y_{S_{k,i}}]\mathbf{M}$
  end for
  for $i = 2$ to $N_{d,k} - 1$ **do**
   $[1, x_{D'_{k,i}}, y_{D'_{k,i}}, x_{D'_{k,i}} y_{D'_{k,i}}] \leftarrow [1, x_{D_{k,i}}, y_{D_{k,i}}, x_{D_{k,i}} y_{D_{k,i}}]\mathbf{M}$
  end for
  Step 3: Find virtual relay placement in $\mathfrak{S}'_k$ to either minimise ED or minimise EB as in [6]: (x'_{R_k}, y'_{R_k}).
  Step 4: Find real relay placement in $\mathfrak{S}_k$:
   $[1, x_{R_k}, y_{R_k}, x_{R_k} y_{R_k}] \leftarrow [1, x'_{R_k}, y'_{R_k}, x'_{R_k} y'_{R_k}]\mathbf{M}^{-1}$
end for

V. Simulation Results

In this section, we present simulation results of the relay placement in a RAWMN using HARQ-IR with PNC protocol. As noted in subsection IV-B, for simplicity, we consider a subset of the RAWMN consisting of two sources $\{\mathcal{S}_1, \mathcal{S}_2\}$, a relay $\mathcal{R}$ and two destinations $\{\mathcal{D}_1, \mathcal{D}_2\}$[3]. All channels experience quasi-static Rayleigh block fading. The data rate of the transmission from $\mathcal{S}_1$ and $\mathcal{S}_2$ is set as $r_1 = r_2 = R = 5$ (bps). The total power consumption of the sources and relay is $P_{\text{total}} = 5$ (W) and the pathloss exponent between a pair of transceiver nodes is $\nu = 3$.

Figures. 1 and 2 sequentially plot the optimal relay positions for minimising ED and minimising EB for various scenarios of power allocation and location of sources and destinations. Specifically, three scenarios of power allocation at sources $\mathcal{S}_1$ and $\mathcal{S}_2$ (i.e. P_1 and P_2) are considered, including $P_1 = P_2$, $P_1 = 2P_2$ and $P_1 = 4P_2$. Here, P_1 is assumed to vary in $[0.1 : 0.1 : 2.4]$ (W). The sources and destinations are assumed to form a geometric shape $\mathfrak{S}$ with the coordinates: $(x_{S_1}, y_{S_1}) = (0, 3)$, $(x_{S_2}, y_{S_2}) = (-1, 0)$, $(x_{D_1}, y_{D_1}) = (4, 4)$ and $(x_{D_2}, y_{D_2}) = (5, -1)$. Instead of finding directly the relay placement, we apply the proposed perspective transformation algorithm (see Algorithm 1) to map $\mathfrak{S}$ to the rectangular shape $\mathfrak{S}'$ having nodes: $(x'_{S_1}, y'_{S_1}) = (0, 1)$, $(x'_{S_2}, y'_{S_2}) = (0, 0)$, $(x'_{D_1}, y'_{D_1}) = (1, 1)$ and $(x'_{D_2}, y'_{D_2}) = (1, 0)$. Applying bisection search method as in [6], we can find the virtual relay positions for minimising ED and minimising EB as shown in Figs. 1(a) and 2(a), respectively. Then, by inverse mapping, the real relay placement for minimising ED and minimising EB in $\mathfrak{S}$ can be easily found as in Figs. 1(b) and 2(b), respectively. It

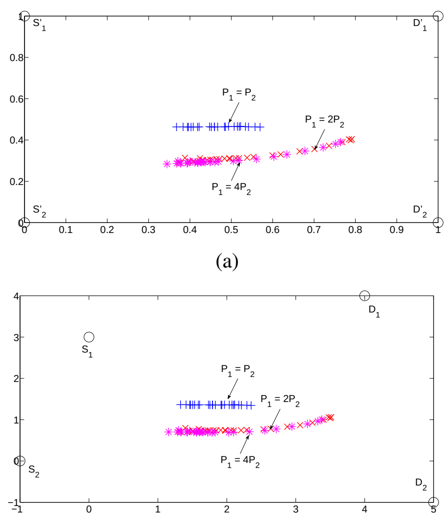
[3]The extension to a larger subset of more-than-two sources and destinations can be straightforwardly obtained using the proposed Algorithm 1.



(a)

(b)

Fig. 1: Optimal relay placement for minimising ED: (a) virtual position and (b) real position.



(a)

(b)

Fig. 2: Optimal relay placement for minimising EB: (a) virtual position and (b) real position.

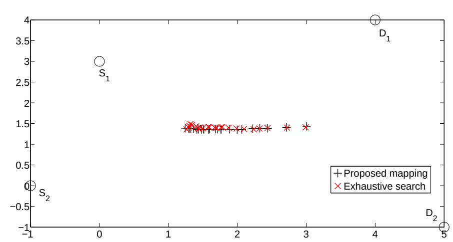


Fig. 3: Relay placement schemes for minimising ED.

can be observed that only mapping matrices are required and the lines connecting nodes in $\mathfrak{S}$ are shown to be maintained as stated in subsection IV-A.

Furthermore, validating the effectiveness of the proposed relay placement, Fig. 3 plots the relay positions for minimising ED with respect to the proposed scheme and the exhaustive search scheme. The simulation parameters are similarly set as those in Fig.1(b). Equal power allocation at the sources is considered and assumed to vary in $[0.1 : 0.1 : 2.4]$ (W). In the exhaustive search, the optimal relay position is found by searching all possible positions in the area covered by $\mathcal{S}_1$, $\mathcal{S}_2$, $\mathcal{D}_1$ and $\mathcal{D}_2$. It can be observed in Fig. 3 that the relay positions with the proposed mapping and exhaustive search schemes are nearly consistent. Further results also show the proposed scheme achieves the similar minimum ED and minimum EB compared to the exhaustive search scheme, however they are omitted due to space limitation. This accordingly means the proposed perspective transformation algorithm achieves a lower complexity while still guaranteeing the optimal relay placement in the irregularly-shaped RAWMN.

VI. Conclusion

In this letter, we have investigated the relay placement problem in irregularly-shaped RAWMNs employing HARQ-IR with PNC protocol. By exploiting perspective transformation in image mapping, an efficient relay placement algorithm has been proposed to find the optimal relay positions for minimising either ED or EB in RAWMNs. The proposed algorithm not only reduces the searching complexity but also allows us to address the problem of irregular and variant shapes of the RAWMNs. For future work, we will analyse the effects of the mobility of nodes as well as the interferences caused by non-intended nodes on the relay placement in practical RAWMNs such as adhoc and sensor networks.

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